Micro C – Spring 2013 – Exam Solutions

1.

(a) First, L dominates M for player 2. Then T dominates L for player 1. No further eliminations are possible.

(b) The remaining game is:

	L	R
T	$7,6$	$3,8$
B	$8,2$	$0,0$

There are two equilibria in pure strategies: (B, L) and (T, R). In a mixed-strategy equilibrium, both players need to be indifferent between their strategies. Let p be the probability that 1 plays T, and q the probability that 2 chooses L. Then

$$\begin{aligned} 7q + 3 - 3q &= 8q \\ q &= \frac{3}{4} \end{aligned}$$

and

$$\begin{aligned} 6p + 2 - 2p &= 8p \\ p &= \frac{1}{2} \end{aligned}$$

so that the Nash Equilibrium in mixed strategies is $\left(\left(\frac{3}{4}, \frac{1}{4}\right), \left(\frac{1}{2}, \frac{1}{2}\right)\right)$ or $(p^*,\, q^*) = \left(\frac{3}{4}, \frac{1}{2}\right)$.

(c) If the game is repeated twice, a strategy for each player must specify 10 actions (one in the first stage, and one after every possible outcome from the first stage). Every combination of 10 actions such that in each stage the players play either (B, L) or (T, R) is a Nash Equilibrium, for example

$$(BBBBBBBBBB,\ LLLLLLLLLL) \text{ or}$$

$$(BTBTBTBTBT,\ LRLRLRLRLR)\,.$$

2.

(a) This violates pareto-optimality, since not all cookies are distributed.

(b) This violates symmetry, since different players get different amounts of cookies for no reason.

3.

(a) The profit functions of the two farmers are

$$\begin{aligned}\pi_A(w_A,\, w_B) &= \frac{2}{3}(w_A + w_B) - \frac{(w_A)^2}{2} \\ \pi_B(w_A,\, w_B) &= \frac{1}{3}(w_A + w_B) - (w_B)^2\,.\end{aligned}$$

The first-order conditions immediately give the optimal work levels for both players

$$\begin{aligned}w_A^* &= \frac{2}{3} \\ w_B^* &= \frac{1}{6}\end{aligned}$$

(b) Total profit is

$$\Pi(w_A,\, w_B) = w_A + w_B - \frac{(w_A)^2}{2} - (w_B)^2$$

which leads (through the first-order conditions) to the socially optimal work levels

$$\begin{aligned}w_A^{SO} &= 1 \\ w_B^{SO} &= \frac{1}{2}\end{aligned}$$

If we use the work levels from (a), w_A^* and w_B^*, we get a total profit of $\frac{7}{12}$. Total profit from w_A^{SO} and w_B^{SO} is $\frac{3}{4}$, or $\frac{9}{12}$, which is larger than $\frac{7}{12}$.

[It is of course not important which notation is used for the solutions from (a) and (b). It is important that the notation is consistent and that the two profits, plus total profits, from (a) and (b) are given.]

(c) If both farmer choose independently, they don't consider that their work also benefits the other farmer. If the farmers maximize total profit, these positive externalities are internalized and the farmers put in more work than in the Nash equilibrium, thus raising total profit.

(d) Farmer B has a profit of $\frac{1}{4}$ in the Nash Equilibrium, and a profit of $\frac{1}{4}$ in the social optimum. (For farmer A, the numbers are $\frac{1}{3}$ and $\frac{1}{2}$, but that is not essential for the argument here.) If farmer A chooses the social optimal level of work, $w_A^{SO} = 1$, farmer B can choose $w_B = \frac{1}{6}$ and get a profit of $\frac{13}{36} > \frac{1}{4}$ in the first round. In the following rounds, he will not be punished for his deviation, since he will get a profit of $\frac{1}{4}$ both in the social optimum and in the Nash Equilibrium. [It is important for the argument to show that B can do better than $\frac{1}{4}$ by his best deviation. The answer is not complete if it simply argues that B gets the same profit from the NE and the SO.]

4.

(a) It is a game of perfect information: All players know the history of the game when it is their move. The strategy sets are

$$\begin{aligned} S_1 &= \{L, R\} \\ S_2 &= \{A, B\} \\ S_3 &= \{C, D\} \end{aligned}$$

(b) The backward-induction outcome is that player 3 chooses C, player 2 chooses A and 1 chooses L. The SPNE is (L, A, C).

(c)

L (Pl. 2 rows, Pl. 3 columns):

	C	D
A	$3, 2, 1$	$0, 0, 0$
B	$2, 1, 1$	$2, 1, 1$

R (Pl. 2 rows, Pl. 3 columns):

	C	D
A	$2, 2, 2$	$2, 2, 2$
B	$2, 2, 2$	$2, 2, 2$

There are four Nash Equilibria that are not subgame-perfect: (L, B, D), (R, A, D), (R, B, C) and (R, B, D).

(d) In each of the NE that are not subgame-perfect, one of the players chooses a strategy that is not a best response in this subgame. So:

- (L, B, D): D is not a best response in the subgame where only player 3 has to choose.
- (R, A, D): D is not a best response in the subgame where only player 3 has to choose.
- (R, B, C): B is not a best response in the subgame which starts at player 2's decision node.

- (R, B, D): B is not a best response in the subgame which starts at player 2's decision node, and D is not a best response in the subgame where only player 3 has to choose.

5.

(a) $((L, R), (d, d), p = 1, q = 0)$. There is no PBE in which t_1 chooses R, and hence no separating PBE with (R, L).

(b) $\left((L, L), (d, u), p = \frac{1}{2}, q \geq \frac{1}{3}\right)$. Since message R is strictly dominated for t_1, requirement 5 says that the receiver should not believe that t_1 would play R with positive probability. Requirement 5 hence requires that $q = 0$, which is mutually exclusive with the pooling equilibrium.